The Snuffle Problem

Petry

for the Heat Conduction in a Thin Annulus

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Definition of the Snuffle Problem Petry

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Based on a former research project (see [1, Section 3.3 on page 110]) we are interested in the additional calculation of the stationary temperature field of a model fluid in a very thin annulus between a housing and a piston. The heat equation for an incompressible Newtonian fluid using axisymmetric cylindrical coordinates is:

$$u\,\frac{\partial T}{\partial r} + w\,\frac{\partial T}{\partial z} = \kappa\,(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2}) + 2\frac{\nu}{c_p}\hat{\varepsilon}^2 \tag{1}$$

with:

$$\hat{\varepsilon}^2 = (\frac{\partial u}{\partial r})^2 + (\frac{\partial w}{\partial z})^2 + \frac{u^2}{r^2} + \frac{1}{2}(\frac{\partial w}{\partial r} + \frac{\partial u}{\partial z})^2\ . \tag{2}$$

Therein, $T = T(r, z)$ is the unknown temperature, u and w are the velocities known from the solution of the fluid-structure interaction problem, $\hat{\varepsilon}$ is the strain tensor, κ is the thermal diffusivity, ν is the kinematic viscosity and c_p is the specific heat capacity. In order to distinguish between quantities in the fluid and quantities in the piston and the housing respectively we denote quantities in the piston and in the housing with an asterisk. Due to $u^* = w^* = 0$ the heat equation is reduced to:

$$\frac{\partial^2 T^*}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}^* + \frac{\partial^2 T}{\partial z^2}^* = 0\ . \tag{3}$$

Both piston and housing consist of steel, while the lubricant in the gap is a model fluid. The material properties for steel and fluid are given in Table 1. Therein, ϱ and ϱ^* are the densities, η is the dynamic viscosity and λ, λ^* are the heat conductivities. Furthermore, it holds:

Table 1. Material properties for steel and fluid.

$\kappa = 91.5 \cdot 10^{-9}\,\mathrm{m^2/s}$	$\lambda = 0.15\,\mathrm{W/m/K}$	$\varrho = 800\,\mathrm{kg/m^3}$	$c_p = 2050\,\mathrm{J/kg/K}$
$\kappa^* = 10.7 \cdot 10^{-6}\,\mathrm{m^2/s}$	$\lambda^* = 42\,\mathrm{W/m/K}$	$\varrho^* = 7850\,\mathrm{kg/m^3}$	$c_p^* = 502\,\mathrm{J/kg/K}$
$\eta = 2 \cdot 10^{-3}\,\mathrm{Pa\,s}$	$\nu = 2.5 \cdot 10^{-6}\,\mathrm{m^2/s}$		

$$\nu = \frac{\eta}{\rho}\ , \qquad \kappa = \frac{\lambda}{\rho\, c_p}\ . \tag{4}$$

On the outer boundaries at $z = 0$ and $r = r_a$ respectively we assume:

$$T = T^* = T_0 := 20℃\ . \tag{5}$$

On the outer boundary at $z = z_e$ we assume:

$$\frac{\partial T}{\partial z} = \frac{\partial T^*}{\partial z} = 0 \; . \tag{6}$$

At the fluid-steel interfaces we postulate the following two conditions:

$$T^* = T \; , \tag{7}$$

$$\lambda^* \frac{\partial T^*}{\partial r} = \lambda \frac{\partial T}{\partial r} \; . \tag{8}$$

Results of the Snuffle Problem Petry by the FDEM Program

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The snuffle problem from Dr. Martin Petry from Bosch deals with the simulation of the temperature of the fluid in a lubrication gap between a housing and a piston. The domain consists of 3 subdomains with different systems of PDEs: piston, housing and fluid. In a former research project (see [1, Section 3.3 on page 110]) we computed the stresses and the displacements w and u in z- and r-direction in the piston and the housing, and the pressure p and the velocities w and u in z- and r-direction of the fluid in the lubrication gap.

By the pressure of 2000 *bar* the lubrication gap widens and changes its form. For the snuffle problem this changed form and the velocities in the lubrication gap are given, and by the solution of the PDE systems we obtain the temperature sequence in piston, housing and, as a matter of particular interest, in the gap.

As there was an error in the value for η/ρ when we solved the fluid-structure interaction problem (the value for η/ρ was too small by a factor of 100), we first had to repeat this computation for the correct value $\eta/\rho = 2.5 \cdot 10^{-2}\ cm^2/s$. As input data for the heat equation we need the velocity components w and u. In order to have small errors for the temperature T we must assure that also w and u have small errors. In order to get a maximum relative error in the 1% range, we needed 641 nodes in radial direction in the lubrication gap, i.e. the finer grid was forced by the errors. For the results and detailed information of the solution of the fluid-structure interaction problem, we refer to [1], Section 3.3. The erratum which has become necessary because of the wrong value of η/ρ, can be found on page 151 of the actual Internet version.

For the actual snuffle problem we want to compute the temperature T for the heat conduction in piston, lubrication gap and housing. In the lubrication gap we use the heat equation for incompressible fluids which is in cylindrical coordinates

$$u\frac{\partial T}{\partial r} + w\frac{\partial T}{\partial z} = \kappa\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2}\right) + 2\frac{\nu}{c_p}\hat{\varepsilon}^2 \qquad (1)$$

with

$$\hat{\varepsilon}^2 = \left(\frac{\partial u}{\partial r}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2 + \frac{u^2}{r^2} + \frac{1}{2}\left(\frac{\partial w}{\partial r} + \frac{\partial u}{\partial z}\right)^2 \ . \qquad (2)$$

Here, $T = T(z, r)$ is the unknown temperature, w, u are the velocities known from the solution of the fluid-structure interaction problem, $\hat{\varepsilon}$ is the strain tensor, κ is the thermal diffusivity and c_p is the specific heat capacity.

In the piston and the housing the heat equation is reduced to

$$\frac{\partial^2 T^*}{\partial r^2} + \frac{1}{r}\frac{\partial T^*}{\partial r} + \frac{\partial^2 T^*}{\partial z^2} = 0 \ . \qquad (3)$$

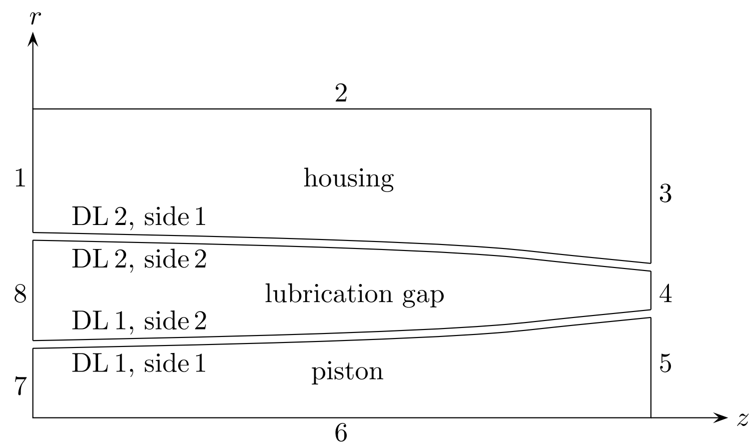

Fig. 1. Illustration of external boundaries and dividing lines (DL) of piston, lubrication gap and housing. The lubrication gap that has a width of a few micrometers is largely blown up.

Fig. 1 shows the 8 external boundaries and the 2 dividing lines (DL) of the domain. In Table 1 we present the boundary conditions for the external boundaries, in Table 2 we show the coupling conditions for the dividing lines, but in both tables we exclude the corners of the subdomains. In Table 2 λ and λ^* are the heat conductivities of the two materials (the

Table 1. Boundary conditions for the external boundaries of Fig. 1, excluding the corners.

Bd.	condition
1	$T^* = T_0 = 20$
2	$T^* = T_0 = 20$
3	$\frac{\partial T^*}{\partial z} = 0$
4	$\frac{\partial T}{\partial z} = 0$
5	$\frac{\partial T^*}{\partial z} = 0$
6	$\frac{\partial T^*}{\partial r} = 0$
7	$T^* = T_0 = 20$
8	$T = T_0 = 20$

Table 2. Coupling conditions for the dividing lines of Fig. 1, excluding the corners.

DL	side	condition
1	1	$T^* = T$
1	2	$\lambda^* \frac{\partial T^*}{\partial r} = \lambda \frac{\partial T}{\partial r}$
2	1	$T^* = T$
2	2	$\lambda^* \frac{\partial T^*}{\partial r} = \lambda \frac{\partial T}{\partial r}$

asterisk denotes housing and piston values).

The boundary conditions at the 12 corners of the 3 subdomains are shown in Table 3. In Table 4 we give the material parameters for the model fluid and steel that we use for the computation.

Table 3. Boundary conditions for the corners of the 3 subdomains.

	Corner			
subdomain	upper left	upper right	lower left	lower right
piston	$T^* = T_0 = 20$	$T^* = T$	$T^* = T_0 = 20$	$\frac{\partial T^*}{\partial z} = 0$
lubrication gap	$T = T_0 = 20$	$\frac{\partial T}{\partial z} = 0$	$T = T_0 = 20$	$\frac{\partial T}{\partial z} = 0$
housing	$T^* = T_0 = 20$	$T^* = T_0 = 20$	$T^* = T_0 = 20$	$T^* = T$

Table 4. Material parameters for the model fluid and steel.

$$\begin{aligned} \lambda &= 0.15\ W/(m \cdot K), & \lambda^* &= 42\ W/(m \cdot K) \\ \rho &= 800\ kg/m^3, & \rho^* &= 7850\ kg/m^3 \\ c_p &= 2050\ J/(kg \cdot K), & c_p^* &= 502\ J/(kg \cdot K) \\ \eta &= 2 \cdot 10^{-3}\ Pa \cdot s \end{aligned}$$

Furthermore, it holds

$$\nu = \frac{\eta}{\rho}, \qquad \kappa = \frac{\lambda}{\rho c_p} \ . \tag{4}$$

For the computations we have used the distributed memory supercomputer HP XC6000 of the University of Karlsruhe with Itanium2 processors, 1.5 GHz, 2-processor nodes with Quadrics interconnect. We computed in parallel on 16 processors. The grid for the 3 subdomains is:

piston:	401 (z-direction)×40 (r-direction),
lubrication gap:	401×641,
housing:	401×81.

The computation time for the master processor 1 is 155 sec. The results of the computation are shown in Table 5 where we present the maximum temperature, the maximum relative estimated error and the mean relative estimated error for the 3 subdomains. We see

Table 5. Maximum temperature, maximum and mean relative estimated error for piston, lubrication gap and housing.

subdomain	T_{max} [°C]	max. rel. est. error	mean rel. est. error
piston	98.2	$0.29 \cdot 10^{-1}$	$0.46 \cdot 10^{-4}$
lubrication gap	98.9	$0.29 \cdot 10^{-1}$	$0.84 \cdot 10^{-3}$
housing	87.8	$0.40 \cdot 10^{-2}$	$0.24 \cdot 10^{-4}$

that the maximum relative errors are about 3% for the piston and the lubrication gap, but errors in the range of the maximum error appear only in a few nodes as the mean relative errors are $0.46 \cdot 10^{-4}$ in the piston and $0.84 \cdot 10^{-3}$ in the lubrication gap. In the housing the errors are even smaller.

Figs. 2–4 show the temperature T in the fluid and T^* in the structure and its error for the 3 subdomains. The temperature increases from 20°C at $z = 0$ to 98.9°C at $z = 4$ cm. From the error pictures at the right side of each figure we can also see that the maximum errors occur only in few nodes.

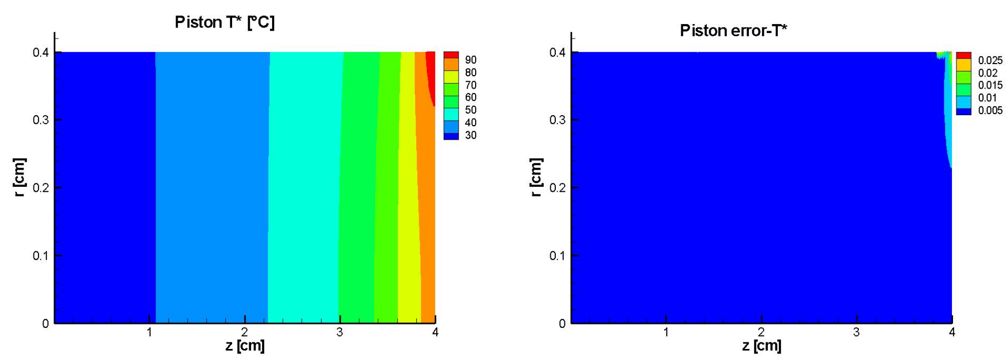

Fig. 2. Contour plot for the temperature T^* and its error in the piston.

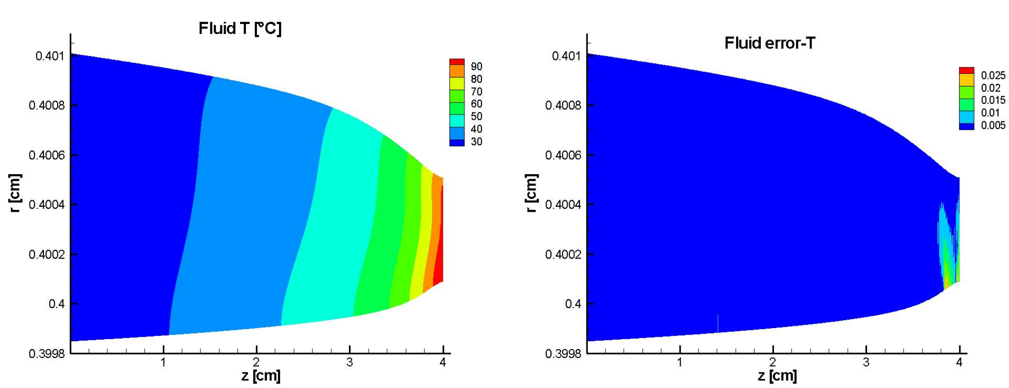

Fig. 3. Contour plot for the temperature T and its error in the lubrication gap.

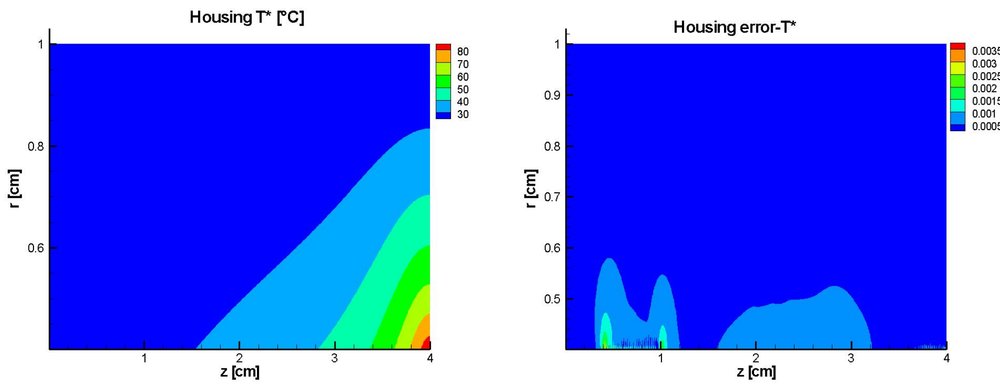

Fig. 4. Contour plot for the temperature T^* and its error in the housing.

References

1. Schönauer, W., Adolph, T., FDEM: The Evolution and Application of the Finite Difference Element Method (FDEM) Program Package for the Solution of Partial Differential Equations, Abschlussbericht des Verbundprojekts FDEM, Universität Karlsruhe, 2005, available at http://www.rz.uni-karlsruhe.de/rz/docs/FDEM/Literatur/fdem.pdf .