

Pseudo-time stepping

Target group

M.Sc. Mathematics

Description of the topic

Context. In fluid mechanical settings, researchers often simulate systems of coupled non-linear partial differential equations, such as the Euler equations, describing a system in steady state. Discretizing these equations results in a system of non-linear equations which must be solved to find a consistent discrete solution vector. To solve this non-linear system, an iterative scheme is needed where, in each iteration, the non-linear system is linearized around the current iteration of the solution vector. A new iteration of the solution vector is then derived from the linear system.

Pseudo-time stepping. If the initial estimate for the solution vector is too far from the true solution of the non-linear system of equations, a classic Newton iteration will often not converge. As an alternative, a common practice by fluid-mechanical engineers is to take an approach called pseudo-time stepping. In this approach, the steady state partial differential equation is replaced by a time-dependent equation. The initial estimate for the solution is then interpreted as a non-equilibrium initial condition to the time-dependent system of PDEs. This time-dependent system is then simulated with an implicit scheme until it reaches a steady state solution.

Challenges. In current practice, this approach is often applied in an ad-hoc way, where the parameters are selected by trial and error until the system converges at a reasonable computational cost. We will investigate how to analyze this approach in a more mathematically rigorous way.

Helpful prerequisites

Helpful prerequisites for working on this topic:

- Optimization theory
- Numerical simulation of PDEs
- Programming: C++ is a bonus, Python is also possible

Exemplary questions

- Can a link between pseudo-time stepping and regularization theory be used to derive good optimization approaches?
- How does pseudo-time stepping compare to other ad-hoc methods used to get solvers to converge?

Contact

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